

## Some Excel 2003 Problems

It should be possible to solve all the problems presented in this handout by using the information given in the various sessions which are issued as separate documents.

### 1. The Fibonacci Series Problem

In the following, each number is the sum of the two immediately preceding numbers. The series is known as the Fibonacci series:

$$1 + 1 + 2 + 3 + 5 + 8 + 13 + \dots$$

Prepare an Excel worksheet to determine two values:

- The first number in the series that is greater than 1000.
- The sum of the series up to and including that number.

### 2. The Multiplication Table Problem

Prepare an Excel worksheet to display the Multiplication Table up to  $12 \times 12$  in the following way:

THE MULTIPLICATION TABLE

|    |    |    |    |    |    |    |    |     |     |     |     |
|----|----|----|----|----|----|----|----|-----|-----|-----|-----|
| 1  | 2  | 3  | 4  | 5  | 6  | 7  | 8  | 9   | 10  | 11  | 12  |
| 2  | 4  | 6  | 8  | 10 | 12 | 14 | 16 | 18  | 20  | 22  | 24  |
| 3  | 6  | 9  | 12 | 15 | 18 | 21 | 24 | 27  | 30  | 33  | 36  |
| 4  | 8  | 12 | 16 | 20 | 24 | 28 | 32 | 36  | 40  | 44  | 48  |
| 5  | 10 | 15 | 20 | 25 | 30 | 35 | 40 | 45  | 50  | 55  | 60  |
| 6  | 12 | 18 | 24 | 30 | 36 | 42 | 48 | 54  | 60  | 66  | 72  |
| 7  | 14 | 21 | 28 | 35 | 42 | 49 | 56 | 63  | 70  | 77  | 84  |
| 8  | 16 | 24 | 32 | 40 | 48 | 56 | 64 | 72  | 80  | 88  | 96  |
| 9  | 18 | 27 | 36 | 45 | 54 | 63 | 72 | 81  | 90  | 99  | 108 |
| 10 | 20 | 30 | 40 | 50 | 60 | 70 | 80 | 90  | 100 | 110 | 120 |
| 11 | 22 | 33 | 44 | 55 | 66 | 77 | 88 | 99  | 110 | 121 | 132 |
| 12 | 24 | 36 | 48 | 60 | 72 | 84 | 96 | 108 | 120 | 132 | 144 |

Assuming that you leave the heading and the immediately-following blank line until last, the row containing 1 2 3 4 ... will initially be row 1. Assuming further that the leftmost column is column A, the formula in cell B2 might naïvely be =A2\*B1. By incorporating two judiciously-placed \$-signs into the formula you can fill downwards and rightwards and complete the entire table in two simple operations.

Adjust the column widths so that the entire table fits onto a sheet of A4 paper arranged portrait fashion.

*Further problems overleaf*

### 3. The Date of Easter Problem (optional)

A convenient algorithm for determining the date of Easter in a given year was devised in 1876 and first appeared in *Butcher's Ecclesiastical Handbook*. The algorithm is valid for all years in the Gregorian calendar.

As presented here, the algorithm makes use of two Excel functions INT and MOD for division of one whole number by another. For example, INT(1990/100) is exactly 19 and MOD(1990,100) is exactly 90. Thus INT throws away the remainder when 1990 is divided by 100 and MOD supplies that remainder. Notice that after INT a stroke separates the numerator and denominator but after MOD a comma separates the two.

Appropriately adapted for Excel, the algorithm is as follows:

1. Let Year be the year
2. Set aa to MOD(Year,19)
3. Set bb to INT(Year/100) and cc to MOD(Year,100)
4. Set dd to INT(bb/4) and ee to MOD(bb,4)
5. Set ff to INT((bb+8)/25)
6. Set gg to INT((bb-ff+1)/3)
7. Set hh to MOD(19\*aa+bb-dd-gg+15,30)
8. Set ii to INT(cc/4) and kk to MOD(cc,4)
9. Set ll to MOD(32+2\*ee+2\*ii-hh-kk,7)
10. Set mm to INT((aa+11\*hh+22\*ll)/451)
11. Set nn to INT((hh+ll-7\*mm+114)/31) and pp to MOD(hh+ll-7\*mm+114,31)
12. The value of nn gives the month (3 for March and 4 for April) and the value of pp+1 gives the day of the month.

Prepare an Excel worksheet which gives the dates of Easter from 1990 to 2010. The first few lines should be as shown below. The years are shown in the column headed Year and the dates of Easter are shown in the next two columns. Thus, in 1990, Easter fell on 15 April. The columns headed aa, bb, etc. show values corresponding to the identifiers used in the algorithm. Two-letter identifiers are used to avoid confusion with the names of the worksheet columns.

#### THE DATE OF EASTER

| Year | Month | Day | aa | bb | cc | dd | ee | ff | gg | hh | ii | kk | ll | mm | nn | pp |
|------|-------|-----|----|----|----|----|----|----|----|----|----|----|----|----|----|----|
| 1990 | 4     | 15  | 14 | 19 | 90 | 4  | 3  | 1  | 6  | 20 | 22 | 2  | 4  | 0  | 4  | 14 |
| 1991 | 3     | 31  | 15 | 19 | 91 | 4  | 3  | 1  | 6  | 9  | 22 | 3  | 0  | 0  | 3  | 30 |
| 1992 | 4     | 19  | 16 | 19 | 92 | 4  | 3  | 1  | 6  | 28 | 23 | 0  | 0  | 0  | 4  | 18 |
| 1993 | 4     | 11  | 17 | 19 | 93 | 4  | 3  | 1  | 6  | 17 | 23 | 1  | 3  | 0  | 4  | 10 |
| 1994 | 4     | 3   | 18 | 19 | 94 | 4  | 3  | 1  | 6  | 6  | 23 | 2  | 6  | 0  | 4  | 2  |
| 1995 | 4     | 16  | 0  | 19 | 95 | 4  | 3  | 1  | 6  | 24 | 23 | 3  | 1  | 0  | 4  | 15 |

*Continues overleaf*

## Notes on Solving the Date-of-Easter Problem

Excel has facilities for naming cells and columns of cells and, by exploiting these features, the amount of typing and the number of errors can be reduced. Proceed as follows:

- Key **Year** into cell A1 and 1990 into cell A2.
- Key **=A2+1** into cell A3 and fill downwards from that cell to cell A7. The years 1990 to 1995 should appear.
- Drag downwards from cell A1 to cell A7, thereby selecting the heading **Year** and all the years themselves.
- Open **Insert** and choose **Name ► Create...** The **Create Names** dialogue box has the **Top row** checkbox set. This indicates that the name **Year** in the top row of the current selection will be taken as a name which can be used in formulae. Click **OK**.
- Key **Month** into cell B1 and **Day** into cell C1 but otherwise leave the **B** and **C** columns blank for the moment.
- Key **aa** into cell D1. Now try out a new feature...
- Key the formula **=MOD(Year,19)** into cell D2. In this formula, the value of **Year** will be taken as the value of whatever is in the **Year** column in the same row. This is the value in cell A2 which is 1990.
- Click the **Enter** box to confirm. The value 14 appears. This is the remainder when 1990 is divided by 19.
- Now fill downwards from cell D2 to cell D7. The other values in the column headed **aa** appear. Check the formulae in the cells D2 to D7; they should all be identical.
- Drag downwards from cell D1 to cell D7, thereby selecting the heading **aa** and the first six cells beneath.
- Open **Insert** and choose **Name ► Create...** The **Create Names** dialogue box again has the **Top row** checkbox set. The name **aa** in the top row now will be taken as a name which can be used in formulae. Click **OK**.
- Proceed in a similar manner for the columns to the right of that headed **aa**. Each time, key in the relevant heading and then the appropriate formula in the cell immediately beneath. This can always be expressed in terms of one or more of the headings already set up. After filling down, remember to **Insert** the new **Name** in the way described.
- Attend to the columns headed **Month** and **Day** after setting up all the other columns.
- Extend the table beyond the year 1995; select the whole of the row representing the 1995 entry and fill downwards another 15 rows. The column **Year** should run to 2010 but the other columns will go awry! The problem is that the naming has been undertaken only down to row 7.
- Select the whole of the worksheet from cell A1 to cell Q22 which should be at the bottom right-hand corner of the working area so far. It is possible to create several names at once...
- Open **Insert** and choose **Name ► Create...** Click **OK** in the **Create Names** dialogue box.
- You will get a sequence of messages, one for each name used so far. Each explains that you are about to replace the definition of an existing name by a new one. Accept all the replacements as valid updates.
- Tidy up by formatting the headings (aligning them right) and, finally, insert two lines at the top and add the main heading. This should be centred over the columns that have been used.

*End of Date-of-Easter Problem*

*Further problems overleaf*

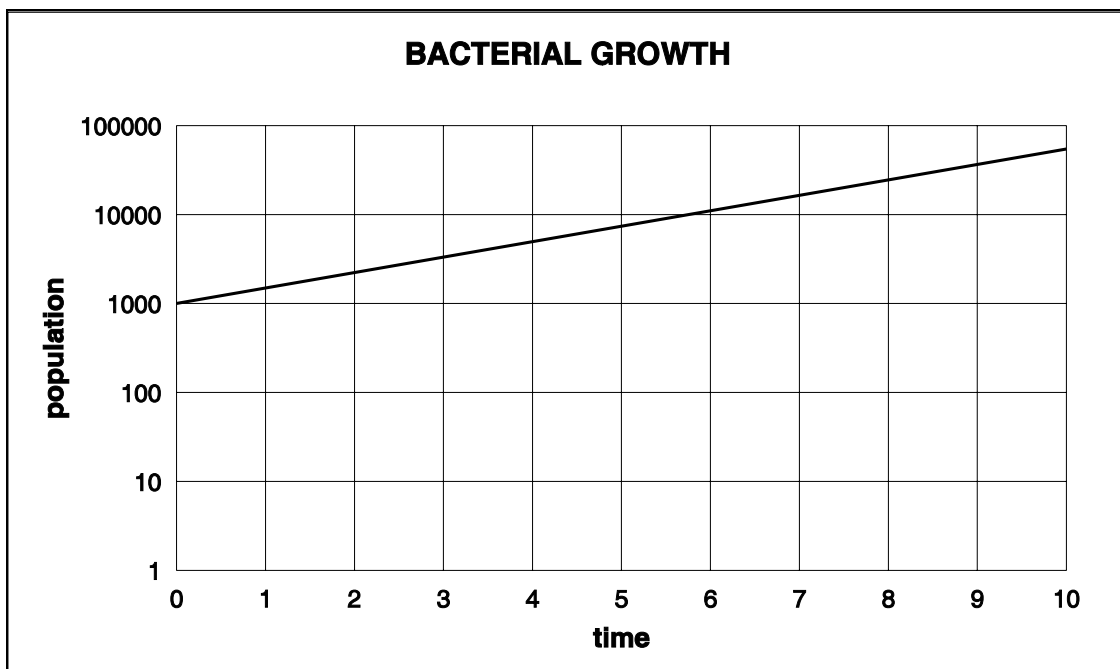
**DO NOT ATTEMPT ANY PROBLEMS FROM THIS POINT  
ONWARD UNTIL YOU HAVE WORKED THROUGH  
“Task 3 — More about Charting in Excel 2003”.**

**4. The Bacterial Growth (and Decay) Problem**

Prepare an Excel worksheet which is laid out roughly as that below. This illustrates a simple model of bacterial growth. The tabulated values show how the population  $n$  varies with time  $t$ . The incorporated chart shows the results as a logarithmic plot.

Two constants  $A$  and  $b$  are assigned the values 1000 and 0.4 respectively using the Insert | Name ► Create... facility in exactly the same way that the constant  $a$  was assigned the value 4 in the Powers.xls example. In the present case, the constant  $A$  is the initial population of bacteria and the constant  $b$  is called the birth rate.

| $t$ | $n$   | $A$  | $b$ |
|-----|-------|------|-----|
| 0   | 1000  | 1000 | 0.4 |
| 1   | 1492  |      |     |
| 2   | 2226  |      |     |
| 3   | 3320  |      |     |
| 4   | 4953  |      |     |
| 5   | 7389  |      |     |
| 6   | 11023 |      |     |
| 7   | 16445 |      |     |
| 8   | 24533 |      |     |
| 9   | 36598 |      |     |
| 10  | 54598 |      |     |



The value 0 at start of the column headed  $t$  and the values of the constants  $A$  and  $b$  are the only numerical values which should be keyed in. The other values in the  $t$  column should be

*Continues overleaf*

generated using **Edit | Fill ▶ Series...** and all the values in the column headed **n** result from a formula. Mathematically, the population  $n$  at time  $t$  is given as:

$$n = A.e^{bt}$$

It is important to note that when  $t = 0$  the value of  $e^{bt} = 1$  so  $n = A$ .

In **Excel** the right-hand side translates into the formula **=A\*EXP(b\*t)** and this formula is in each of the eleven cells in the column headed **n**. It need be keyed in only once of course.

This formula depends not only on **A** and **b** being defined but also on **t** (which relates to an entire column) being defined as well...

The procedure for defining **t** is as follows:

- Key in the heading **t** (and the other three headings as well of course).
- Using the **Edit | Fill ▶ Series...** facility, set up the column series 0,1,2,... 10.
- Select the entire series, including the heading **t**. Open **Insert** and choose **Name ▶ Create...**  
The **Create Names** dialogue box has the **Top row** checkbox set. This indicates that the name **t** in the top row of the current selection will be taken as a name which can be used in formulae. Click **OK**.

Other notes:

- The population values will, by default, be shown to two decimal places and integer values are more appropriate. Use the **Format | Cells...** facility, select **Number** and set the **Decimal places** to 0.
- The vertical axis will need formatting carefully. Format the **Value (Y) axis scale** so that the **Minimum** is 1 and the **Maximum** has the **Auto** checkbox ticked. The **Major unit** should be 10. Ensure that the **Logarithmic scale** checkbox is ticked.

When the worksheet has been set up, it serves as a simple model of bacterial growth. Try the following experiments:

1. Change initial population **A** from 1000 to 500 and then to 2000. Note the consequential changes to the population at time 10.
2. Change initial population **A** back to 1000 but change the birth rate **b** from 0.4 to 0.2 and then to 0.8 and notice the new consequential changes to the population at time 10. A small change in **b** results in a huge change in the population at time 10.
3. Negative values of **b** correspond to decay. This can happen not only to populations of bacteria but also to radioactivity and to concentrations of therapeutic drugs after administration. Change **b** to -0.2, -0.4 and -0.8.

*End of Bacterial Growth (and Decay) Problem*

*Further problems on following pages.*

### 5. The Star and Circle Problem (optional)

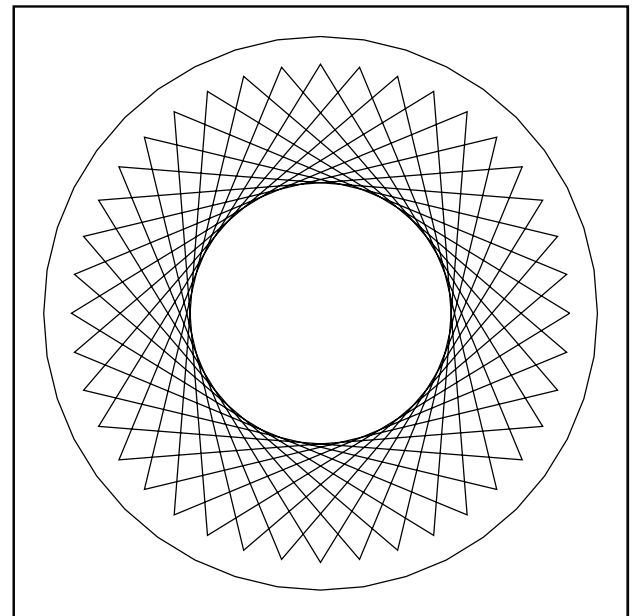
Prepare an Excel worksheet which incorporates an embedded chart in which two data series are plotted, a star with 40-points inside a 40-sided polygon which approximates to a circle.

The worksheet should begin as shown below. The solution is parameterised by two values  $a$  and  $n$ . Here  $a$  is the ratio of the diameter of the star to that of the circle and, in the star, each point is joined to that  $n$  points further round. To be interesting,  $n$  and 40 should be co-prime (have no common factors).

#### STAR AND CIRCLE

| t     | cos(t) | sin(t) | a*cos(nt) | a*sin(nt) |
|-------|--------|--------|-----------|-----------|
| 0.000 | 1.00   | 0.00   | 0.90      | 0.00      |
| 0.157 | 0.99   | 0.16   | -0.41     | 0.80      |
| 0.314 | 0.95   | 0.31   | -0.53     | -0.73     |
| 0.471 | 0.89   | 0.45   | 0.89      | -0.14     |
| 0.628 | 0.81   | 0.59   | -0.28     | 0.86      |
| 0.785 | 0.71   | 0.71   | -0.64     | -0.64     |
| 0.942 | 0.59   | 0.81   | 0.86      | -0.28     |
| 1.100 | 0.45   | 0.89   | -0.14     | 0.89      |
| 1.257 | 0.31   | 0.95   | -0.73     | -0.53     |
| 1.414 | 0.16   | 0.99   | 0.80      | -0.41     |
| 1.571 | 0.00   | 1.00   | 0.00      | 0.90      |
| 1.728 | -0.16  | 0.99   | -0.80     | -0.41     |
| 1.885 | -0.31  | 0.95   | 0.73      | -0.53     |
| 2.042 | -0.45  | 0.89   | 0.14      | 0.89      |
| 2.199 | -0.59  | 0.81   | -0.86     | -0.28     |
| 2.356 | -0.71  | 0.71   | 0.64      | -0.64     |
| 2.513 | -0.81  | 0.59   | 0.28      | 0.86      |
| 2.670 | -0.89  | 0.45   | -0.89     | -0.14     |
| 2.827 | -0.95  | 0.31   | 0.53      | -0.73     |
| 2.985 | -0.99  | 0.16   | 0.41      | 0.80      |
| 3.142 | -1.00  | 0.00   | -0.90     | 0.00      |
| 3.299 | -0.99  | -0.16  | 0.41      | -0.80     |
| 3.456 | -0.95  | -0.31  | 0.53      | 0.73      |
| 3.613 | -0.89  | -0.45  | -0.89     | 0.14      |

a 0.9                      n 13



## 6. The Friday 13th Problem (optional)

It is of interest to note that the 13th of a month is more likely to fall on a Friday than on any other day. For this assertion to be true the Leap Year Cycle of 400 years has to be an integral number of weeks *and* within the cycle the 13ths have to fall on Fridays more often than on other days.

Prepare an Excel worksheet which demonstrates this. The four corners of one possible solution are shown below. This makes use of the fact that 1 January 1900 was a Monday and counts that date as day 1 of a Leap Year cycle. All 4800 thirteenths that lie between 1 January 1900 and 31 December 2299 are inspected. It is left to the reader to check that the total number of days in 400 years is a multiple of 7.

If you are unfamiliar with the Excel functions MOD, IF, OR and AND you should use the Excel help feature and search facility to find out how to use these functions.

In the context of the present problem, note that if...

$$\text{OR}(\text{AND}(\text{MOD}(n,4) = 0, \text{MOD}(n,100) <> 0), \text{MOD}(n,400) = 0)$$

then year n is a leap year.

### THE FRIDAY 13THS PROBLEM

### Thirteenths Counts

| Year | Jan    | Feb    | Mar    | Apr    | ... | Dec    | Sun | Mon | Tues | Wed | Thur | Fri | Sat |
|------|--------|--------|--------|--------|-----|--------|-----|-----|------|-----|------|-----|-----|
| 1900 | 13     | 44     | 72     | 103    | ... | 347    | 1   | 1   | 3    | 1   | 2    | 2   | 2   |
| 1901 | 378    | 409    | 437    | 468    | ... | 712    | 2   | 1   | 1    | 3   | 1    | 2   | 2   |
| 1902 | 743    | 774    | 802    | 833    | ... | 1077   | 2   | 2   | 1    | 1   | 3    | 1   | 2   |
| 1903 | 1108   | 1139   | 1167   | 1198   | ... | 1442   | 2   | 2   | 2    | 1   | 1    | 3   | 1   |
| 1904 | 1473   | 1504   | 1533   | 1564   | ... | 1808   | 2   | 1   | 2    | 3   | 1    | 1   | 2   |
| 1905 | 1839   | 1870   | 1898   | 1929   | ... | 2173   | 1   | 3   | 1    | 2   | 2    | 2   | 1   |
| .    | .      | .      | .      | .      | ... | .      | .   | .   | .    | .   | .    | .   | .   |
| .    | .      | .      | .      | .      | ... | .      | .   | .   | .    | .   | .    | .   | .   |
| 2294 | 143919 | 143950 | 143978 | 144009 | ... | 144253 | 1   | 1   | 3    | 1   | 2    | 2   | 2   |
| 2295 | 144284 | 144315 | 144343 | 144374 | ... | 144618 | 2   | 1   | 1    | 3   | 1    | 2   | 2   |
| 2296 | 144649 | 144680 | 144709 | 144740 | ... | 144984 | 2   | 3   | 1    | 1   | 2    | 2   | 1   |
| 2297 | 145015 | 145046 | 145074 | 145105 | ... | 145349 | 1   | 2   | 2    | 2   | 1    | 1   | 3   |
| 2298 | 145380 | 145411 | 145439 | 145470 | ... | 145714 | 3   | 1   | 2    | 2   | 2    | 1   | 1   |
| 2299 | 145745 | 145776 | 145804 | 145835 | ... | 146079 | 1   | 3   | 1    | 2   | 2    | 2   | 1   |
|      |        |        |        |        |     |        | 687 | 685 | 685  | 687 | 684  | 688 | 684 |

Total number of days in 400 years is: 146097