Task 4 — A Predator–Prey Model

This session presents a slightly-simplified model of a predator–prey system. An outline explanation of the underlying mathematics is given on pages 4.3 and 4.4. It is not necessary to delve too deeply into the mathematical details but the result is a good illustration of how a spreadsheet can be used to help gain an understanding of a biological process.

Outline Requirements

1. Log in to a MCS workstation.

2. Set up, in the Maths folder, an Excel file PredPrey.xls which contains a single worksheet with an embedded chart whose appearance is roughly as shown overleaf.

3. Arrange for the worksheet to be printed in landscape rather than the portrait default.

4. Print just the first sheet and show the hard-copy to a demonstrator.

5. Ask for the Assessed Exercise Taster and work through that.

Check-List

The demonstrator will have the following questions in mind when inspecting your work:

1. Is there a main heading?

2. Is the worksheet printed in landscape rather than portrait?

3. Are the numerical values formatted as in the worksheet overleaf?

4. Have you successfully incorporated the second data series into the chart and arranged to have a Secondary axis on the right-hand side?

5. Have you arranged the legend at the top of the chart?

6. **Is there a footer showing your name and College?**

7. What were the results of the experiments suggested at the end? [Page 4.6]

Herbivores versus Carnivores – A Prey and Predator Model

Equations:

Let h(y) = number of herbivores in year y

Let c(y) = number of carnivores in year y

$h(y+1) = h(y) + b*(hmax-h(y))*h(y) - k*h(y)*c(y)$
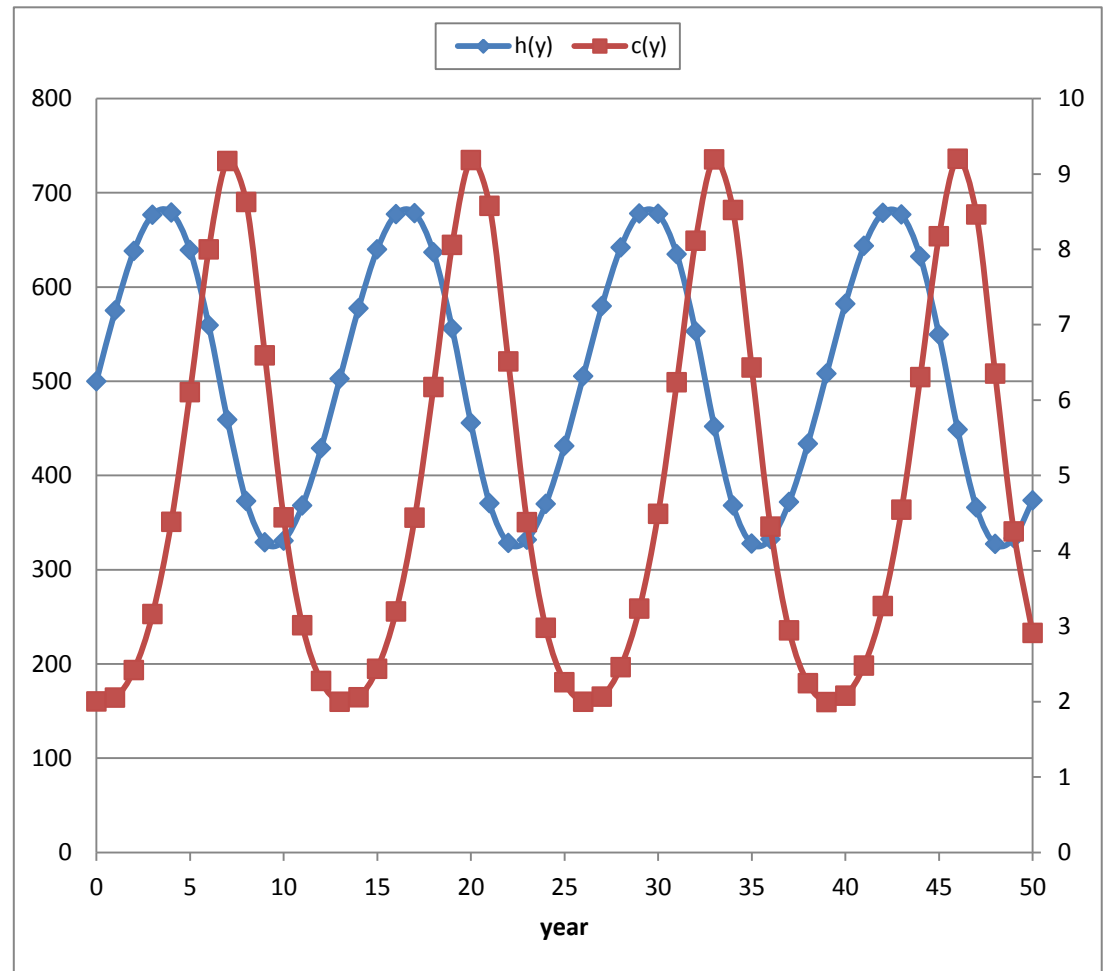
$c(y+1) = q*h(y)*c(y)$

where:

hmax = maximum sustainable number of herbivores

b = herbivore birth rate

k = herbivore kill rate

q = efficiency of use of prey

h0	hmax	c0	b	k	q
500	1000	2	0.0005	0.05	0.00205

y	h(y)	c(y)
0	500.00	2.00
1	575.00	2.05
2	638.25	2.42
3	676.58	3.16
4	679.03	4.39
5	639.12	6.10
6	559.37	8.00
7	458.92	9.17
8	372.63	8.63
9	328.76	6.59
10	330.76	4.44
11	367.97	3.01
12	428.84	2.27
13	502.59	2.00
14	577.39	2.06
.	.	.
.	.	.
.	.	.
50	373.48	2.91

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Task 4 — A Predator–Prey Model

Synopsis: The Microsoft Excel Spreadsheet application. Simple mathematical models.

Briefing

Mathematical models of biological systems are of enormous importance. At their best, they make it possible to conduct elaborate experiments quickly and cheaply and can reduce the need for conducting tests on animals.

Predator–prey models are a good illustration. Many such models have been proposed and some of the more elaborate ones convincingly explain observations made in the field.

In a very simple case where there are unlimited resources and no predators, the prey population will grow in much the same way as the bacteria population of problem 4. Even the primitive model in the worksheet opposite can readily illustrate the simple case: just set the number of carnivores to zero.

The description of the mathematical model given here begins with the very simple case and steadily refines the model. Some suggestions for experimenting with the finished model are presented later.

Note that the population values given in the worksheet are shown to two decimal places. Although it makes more sense to show the values as integers (as in the Bacterial Growth problem) this has not been done. The justification is that there are periods where the predator population changes very slowly so it is useful to see what is going on to the right of the decimal point.

Notation used

It is unfortunate that both the word predator and the word prey begin with p and to avoid confusion the letters h and c are used, standing for herbivore and carnivore.

Additionally, y stands for the year and $h(y)$ and $c(y)$ are, respectively, the numbers of herbivores and carnivores at the start of year y.

The mathematical model proceeds one year at a time, starting with $h0$ herbivores and $c0$ carnivores where $h0 = 500$ and $c0 = 2$.

The model assumes a closed environment, such as an island, which can sustain only a limited number of herbivores even if there are no carnivores intent on eating them. This limited number is $hmax$. In the worksheet its value is taken as 1000.

The herbivore population — no carnivores and unlimited resources

Suppose there are no carnivores and unlimited resources. The *increase* in the number of herbivores during year y is simply proportional to $h(y)$ so that at the start of year $y + 1$ one can say:

$$h(y + 1) = h(y) + b.h(y)$$

In words: the number at the start of the new year is the number at the start of the old year plus some increase. Note that b is some constant representing the birth rate (though, strictly, it represents the excess of birth rate over death rate).

The herbivore population — no carnivores but limited resources

With finite resources the relationship has to be modified. The constant *b* is multiplied by a factor (*hmax* - *h*(*y*)) as in:

$$h(y + 1) = h(y) + b.(hmax - h(y)).h(y)$$

This new factor is positive if the number of herbivores *h*(*y*) is less than the sustainable maximum *hmax* and the population grows. If the number of herbivores *h*(*y*) is greater than *hmax* the factor is negative and the population shrinks.

The herbivore population — carnivores introduced

If carnivores are introduced, the relationship has to be modified further. A quantity has to be subtracted to reflect the number of herbivores being eaten. The final relationship is:

$$h(y + 1) = h(y) + b \times (hmax - h(y)) \times h(y) - k \times h(y) \times c(y) \tag{1}$$

The number of herbivores killed each year is proportional both to the number of herbivores *h*(*y*) and to the number of carnivores *c*(*y*). Note that *k* is some constant representing the kill rate.
In words: the number at the start of the new year is the number at the start of the old year plus some increase due to births but less some decrease due to killing.

The carnivore population

Of course, the number of carnivores changes too. The number of carnivores at the start of year *y* is *c*(*y*). The number at the start of year *y* + 1 is proportional both to the number of herbivores available for eating and to the number of carnivores doing the killing. The relationship used in the model is:

$$c(y + 1) = q \times h(y) \times c(y) \tag{2}$$

Note that *q* is another constant. It is known as the efficiency of use of prey.

Guided Practical Session

- Log in to a MCS workstation.

- Close the Message of the Day window.

- Start Excel:

    | All Programs | Spreadsheets Maths and Statistics | Microsoft Excel 2010

Initialisation

As in the Bacterial Growth problem, several named cells are used. Three names refer to the starting value *h*0, to the maximum sustainable number of herbivores in the absence of carnivores *hmax*, and to the starting value *c*0.

Three other names refer to the constants *b*, *k* and *q*. Look at the worksheet and proceed as follows:

- Key in the introductory text at the head of the worksheet.

- Key in the six names, right-align them and make them bold. Key the six values, 500, 1000, 2, 0.0005, 0.05 and 0.00205 into the cells immediately beneath the names. Then use the

Create from Selection facility (in the Defined Names group of the Formulas tab) to associate the names with the values.

The main columns

- Next key in the three headings corresponding to y, $h(y)$ and $c(y)$. Right-align them and make them bold. The value y runs from 0 to 50 and the values of $h(y)$ and $c(y)$ are shown for the start of each of years 0 to 50. Key in 0 for the initial year and key in =h0 and =c0 as the initial values for the numbers of herbivores and carnivores respectively.

- Key appropriate formulae into the three cells underneath the initial values. Assuming the headings are in columns A, B and C and in row 17 and that the initial values are in row 18, the formulae will be:

  =A18+1
  =B18+b*(hmax-B18)*B18-k*B18*C18   from (1)
  =q*B18*C18   from (2)

- Fill down 50 rows so that the model runs to year 50. Check that the values are as in the worksheet on page 4.2. Adjust the formatting to correspond to that used in the worksheet.

The chart

- Set up an XY (Scatter) chart (of sub-type Scatter with Smooth Lines and Markers) which plots $h(y)$ against y. Delete the chart title and label the horizontal axis year. In the Plot Area set the Border Color to No line and the Fill to No fill. Ensure that the horizontal axis scale runs from 0 to 50. Remove the tick marks from the vertical axis and ensure that the labels are shown as integers like 500 and not as 500.00 (in the Format Axis dialogue box, click on Number in the left hand column and change the number of Decimal places: in the right hand column).

- Add a second data series for $c(y)$ to the chart. (Click on the Select Data button in the Data group on the Design tab, then click the Add button under Legend Entries (Series) to bring up the Edit Series dialogue box. Assuming the headings for the model's data are in columns A, B and C of row 17 of worksheet Sheet1, with the data immediately beneath, this new series will have a Series name: of =Sheet1!$C$17, Series X values: of =Sheet1!$A$18:$A$68, and Series Y values: of =Sheet1!$C$18:$C$68.) The new data series will barely rise from the horizontal axis because the vertical axis extends from 0 to 800 or so whereas the values in the series range from about 2 to 10, so...

- Right-click the second data series and choose Format Data Series… from the sub-menu to bring up the Format Data Series dialogue box. Click on Series Options in the left hand column of the dialogue box and choose the Secondary axis option under Plot Series On in the right hand column. Click Close. A new axis appears at the right. Again ensure that the labels are shown as integers like 5 and not 5.00.

- Tidy up the chart so that is resembles the one on page 4.2. In particular, move the legend to the top of the chart. (Right-click the legend and choose Format Legend… from the sub-menu to bring up the Format Legend dialogue box. Click on Legend Options in the left hand column of the dialogue box and choose the Top option under Legend Position in the right hand column. Click Close.) If necessary, manipulate the legend so that the two keys shown are side-by-side rather than one above the other.

- Arrange for the chart to fill most of the area occupied by the cells in the range G12 to N36.

- Study the chart. Note how the numbers of herbivores and carnivores fluctuate. When there are very few carnivores, the herbivore population increases. Subject to a delay, this results in a growth of the carnivore population but the carnivores eat too many herbivores. The result is a drop in the number of herbivores followed by a drop in the number of carnivores.

More tidying-up

- Select a cell in the worksheet, outside the chart.

- Click on the word Page Layout at the top of the Ribbon to bring up the Page Layout tab. Click the small arrow (⬜) in the lower right corner of the Page Setup group:

…to bring up the Page Setup tab dialogue box and set up the whole page ready for printing. As usual, check via Print the Backstage view that the overall appearance is satisfactory. This should be as on page 4.2. To arrange for the worksheet to be printed in landscape you will need to use the Page tab of the Page Setup tab dialogue box.

Some experiments

- Set $c0 = 0$ so there are no carnivores. Note how the number of herbivores rises to 1000 and stays constant.

- Set $h0 = 1200$ so there are more herbivores than can be sustained. Note how the number of herbivores falls to 1000 and stays constant.

- Restore the original values of $h0$ and $c0$. Try modifying b, k and q. Note that the model can be very sensitive to small changes in one of these constants but that it is less sensitive to changes in the other two.

- Can you find values which lead to extinction?

- What are the principal inadequacies of this model?

Acknowledgement